**Ancient Indian Mathematics and Astronomy**

The impression that science started only Europe was deeply embedded in the minds of educated people all over the world until recently. The alchemists of Arab countries were occasionally mentioned but there was very little reference to India and China. But thanks to studies of Indian National science Academy and other learned bodies, it is becoming clear that India has consistently been a scientific country, right from the ancient to modern times. And it is now universally acknowledged that much of the mathematical knowledge originated in India and moved from East to West.

But what is actually meant by the term ancient?

An idea about the about the time span would be very useful in this context, over which period the scientific knowledge in various streams existed in India.

1. c 3000- c 1300 > Indus Valley Civilisation

1. c 1500bce-c1100bce. > Early Vedic period

3. c1100bce – c500bce.> Later Vedic period

4. c400bce - c200 ce .> Jain period

5. c 400ce - c1600 ce. > Classical period

[bce- before common era, ce –common era]

**Vedas:** They are the religious scriptures of the Hindus. They were the truths revealed to the sages and were orally transmitted through their disciples. Later they were compiled by sage ***Krsna Dvaipayana*** into four divisions :**Rg, Sam, Yajur** and **Atharva.**

Each Veda in its turn was divided into four parts:

1. **Samhitas** – contain the Mantras like concise formulas ,in verse forms
2. **Brahmanas**-contain the explanation of the mantras to be performed during sacrificial rituals
3. **Aranyakas-** contains questions about self exploration
4. **Upanisadas-** knowledge about knowing the Supreme self and the relation between one’s own self and the supreme self.

**All the four Vedas were written in Sanskrit.**

**Mathematics in Vedic period:**

Two branches of Mathematics that were found in the Vedic period are ***Sulva*(**geometry ) and ***Jyotisa*(** Astronomy).This is because during the Vedic period ,Sacrifice(***Jajnas*** )was the chief religious avocation of the people .)This they have to do on sacrificial alters (***jajnabedikas*)** of specific size and shape and also on a specific date of the year and at a specific time of the day. They believed that any alteration in the size and shape of the alter and time and date of the sacrifice would nullify the object of the ritual and even might lead to adverse effects.

The term ***ganita,*** meaning the science of calculations occurs frequently in Vedic literatures. At that time, the term ***ganita*** included arithmetic, astronomy and algebra.

1. **Arithmetic in Vedic period:**

**Use of large numbers was known:** references of numbers up to 10^10 were found in ***Yajurveda Samhita.***

There were specific names for

**(A) Nine numerals:**

***Eka(1),dvi(2),tri(3), catur(4), panca(5), sat(6), sapta(7), asta(8)*** and ***nava(9)***

**(B) Multiples of ten:**

***Dasa(10), Vimsati(20), Trimsat(30), Catvarinsat(40), Pancasat(50), Sasti(60), Saptati(70),***

***Asiti(80), Navati(90)***

**(C)Powers of ten up to**

***Eka-, Dasa-, Sata-, Sahasra-Ayuta- Niyuta -, Prayuta-, Arbuda-, Nyarbuda- Samudra- ,Madhya- Anta- Paradha-.***

Thus any number can be expressed verbally by the nine word numerals and names indicating multiples and powers of ten.

**The concept of decimal system originated in India.** The decimal notation derives its power from two strokes of genius.

1.The concept of place-value and

2.notion of zero as a number.

The place-value principle assigns to each digit in a number, a certain value, by virtue of its position in that number. The place at extreme right has a place-value one; the next place has a value ten, the next hundred, and so on. Thus each place-value is a certain power of 10.

In early layers of Vedic literature, instances of decimal place-value notation appear as *sapta satani vinsatih(720) or sahasrni sata dasa(1110).*

The concept of Zero (0) appeared to the Vedic people as the tenth numeral. It was denoted by the term ***Sunya*** which means void*.*

The journey of ***Sunya*** to ***Zero.***

***Sunya***(Sanskrit)>>>***Sifr***(Arabic)>>>***Cifra(***German)>>>***Zephirum(***Latin)>>>***Zero(***English)

2.**Geometry in the Vedic period:**

***Sulva Sutra(***Aphorisms of the chords):propose constructions of various rectilinear figures and their different combinations.

The mathematician ***Budhayana’s(8th Century bce) Sulva Sutra*** contain

1. Famous Sutra : “**The rope stretched along the length of a rectangle makes an area which the vertical and horizontal sides make together”-** concept very much similar to that of famous theorem that goes by the name **Pythagoras Theorem(c540bce).**
2. Examples of ***Pythagorean*** triples such as (3,4,5) ,(5,12,13), (8,15,17) and so on.
3. Finding the value of square root of 2.
4. Theorems regarding circling of a square
5. Theorems showing diagonals of a rectangle intersect each other, diagonals of a rhombus bisect each other at right angles, area of a square formed by joining the middle points of a square is half its original value, and many others

The other mathematicians ***Apastamba*** and ***Katyayana*** also gave ***Sulva Sutras*** (theorems) corresponding to Pythagoras in almost identical terms.

So we see that the theorem credited to ***Pythagoras*** (**c540bce)** was known to Vedic people at a much earlier date.

**3.Algebra at Vedic period:**

***Panini(c520bce -c460bce)***the Sanskrit Grammarian whose works include early use of **Booleanlogic** and concept of **null operator.**

***Pingala(c300bce-c200bce)*** the musical theorist, authored ***Chadas Shasta,*** a Sanskrit treatise on prosody. He worked on syllabic combinations which gave rise to the ideas corresponding to ***Pascal’s triangle*** (**1965 ce)** which he termed as ***Meru Prastāra.***

[It is important to note that the ***Meru Prastāra***  method is set forth in connection with the problem of determining the number of combinations of a given number of syllables in which a short or the long sound in a *pada* with all the syllables may occur once, twice ,etc up to the total number of syllables. In the following expositions, a and b represent the short and long sounds, and 1,2,3 etc the number of syllables. For example, in the case of a metre with four syllables, a *pada* may contain all the four short sounds(),three short sounds and a long(),two short and two long sounds(), one short and three long sounds(a ),or all the for long sounds().

|  |  |
| --- | --- |
| 1 | |
| 1 | | | | 1 | | |
| 1 | | | 2 | | | | 1 | | |
| 1 | | | 3 | | | | 3 | | | 1 | |
| 1 | | 4 | | | | 6 | | | 4 | | | 1 | |

1(a) +1(b). =

1() + 2(ab) +1(). =

1() +3(b) +3(a) +1() =

1() + 4(b)+6() +4(a) +1(). =( ]

Concepts of ***Binomial co-efficient*** and ***Fibonacci numbers(Maatra Meru*** was his terminology) was there with him, much before the ***Pascal’s triangle***  with its full implication for the expansion of a binomial power series.

***4.*Commerce in Vedic age:**

***Kautilya (4th century bce) ,***author of ***Arthasastra.*** His book describes a sophisticated system of book keeping for *income* with dates and times,; a ledger for *expenditure* and a ledger for *balances.* There are chapters on *auditing, insurance, current vs differed receipts, fixed vs variable cost.* Theseled to concept of ***negative numbers.***

**Jain period:**

Jain Mathematicians freed Mathematics from the religious and ritualistic constraints of the Vedic age. They devised notations for powers and exponents of numbers and so could define simple algebraic equations.(**Beejganita samikaran).**Some important workers of that time are:

**Bhadrabahu(298bce) :**authored astronomical text

**Yativrisham Acharya(176bce):** worked on mathematical text

**Umasvati(150bce)**:author of philosophy and metaphysical texts.

**The classical age**

Mathematics in this period broadened into various branches. Some stalwarts in mathematics of this era are:

**Aryabhata, Varahamihira, Brahmagupta, BhaskaraI, Mahavira, Bhaskara II, Madhava, Shridhar, Nilakanta, Aryabhata II, Manjula, Narayana, Bhattahari.**

Their contributions in mathematics spread to Asia, Middle East and Europe.

**Some texts of the classical age**

**Surya Siddhanta(c400ce),** authorship unknown, contain roots of modern trigonometry.

**Aryabhatiya** written by **Ayarbhata(476-550)ce.** Here fundamental principles of Mathematics are written in 332 shlokas which contains:

Quadratic equations and its solutions

Simultaneous equations and their solutions

Value of pi, correct to 4 decimal places

Definition of sine (jya), cosine(Kojya), and versions (utkrama jya)

Tables of sine, cosine and versions values, in 3.75 degree intervals from 0 to 90 degrees, up to 4 decimal place accuracy.

Spherical trigonometry

Accurate calculations of astronomical constants

**Pancha Siddhanta** by **Varahamihira(505-587)ce.** Here the author studied

Sine and cosine tables’ up to 4 decimal places accuracy

Coined the following formulas of Trigonometry

(Sinx)^2 +(cosx)^2 =1

1. Cos2x)/2=(sinx)^2

**Brahma Sphuta Siddhanta** by **Brahmagupta (628ce)**

Here two fields of Mathematics **Algebra** and **Arithmetic** are treated which later

emerged into two fields ***Bija-Ganitaa*** and ***Pati-Ganita.*** His works include:

Area of cyclic quadrilateral

Area of rational triangle

Solution of quadratic equations

Finding integral solution of equation, which was later named after Pell.

[Example: Find integers such that X^2 -92y^2=1.

The solution provided by **Brahmagupta** was x=1151 & y=120.]

**Bhaskara I(600-680, Virasen(8th century), Mahavira Acharya (800-870), Shridhar**

**Acharya(870-930), Manjula(10th century)** are some of the important Mathematicians who made substantial contributions in the field of Mathematics from 6th through 10th century. The most significant contribution was made by

**Bhaskara II (1114-1185)ce,** whose contributions in the field of mathematics as well as astronomy is equally notable .His contributions were transmitted to Middle and Europe. He made many important contributions in the following fields:

Interest calculations

Arithmetic and Geometric Progression

Plane Geometry

Solid Geometry

Indeterminate Cubic and quartile equation

Indeterminate higher order polynomials

He also

Gave a proof for division by zero yields infinity

Recognised that positive numbers have two square roots

Gave a proof of the Pythagoras theorem

Conceived differential equation

Stated Rolle’s theorem ,a special case of mean value theorem

Computed value of pi, correct to 5decimal places

Calculated the length of Earth’s revolution around the Sun to 9 decimal places

Developed spherical geometry

Developed trigonometric formulas

sin(a+b)= sina cosb+cosa sinb

sin(a-b)=sina cosb-cosa sinb

**Astronomy in ancient India**

**Astronomy in the Vedic age(c1500bce-c500bce.**

Oldest available treatise on Astronomy is ***Vedanga jyotisa(c 1300bce)*** by **sage *Lagadha.***

The extant text is dated to the final centuries BCE

***Vedanga Jyotisa*** is one of the six auxiliary disciplines whose knowledge helped in understanding the 4 Vedas.(They are ***Siksha, Kalpa, Jyotisa, Chanda, Vyakarana and Nirukta.)***

Notion of twenty seven constellations, twelve signs of Zodiac, Solar and Lunar Eclipses were known and dealed with in that treatise .

**Truths of Astronomy found in other Vedas:**

In ***Rg-Veda*** there are references for axial rotation and heliocentric revolution of the Earth.

***Aiteriya Brahmana*,*Chandogya Upanisada***and ***Visnu Puran*** have shlokas which mean

**“In reality,the Sun never rises nor sets”**

**Skanda Puran** states that the Earth is revolving like a ***bhramarika*** or a spinning top. The concept of precession movement was also there and was known as ***Ayan chalan***.

A revered name in ancient Indian Astronomy is ***Vrddha Garg.***He is frequently referred to in the post Vedic treatise and in the epic *Mahabharata.*

**Astronomy in the classical age (400ce-1600ce)**

**Some noted astronomers:**

**1.Aryabhata(5th century ce)** : pioneer in modern concepts of astronomy. His work ***Aryabhatiyam*** influenced Indian Astronomy and Mathematics for more that 1000 years.

He was a meticulous observer and experimenter. His record of many astronomical constants are still in good agreement with the modern calculations.

Some of his highlights:

1. Principle of Axial rotation of Earth.He explained thus the apparent westward motion of the stars.

2. Correct observation of difference between duration of solar day and sidereal day

[Solar day is the time taken by sun to go round the earth once relative to a terrestrial observer. i.e. time interval between two successive sunrises. Sidereal day is the time taken by earth to rotate round its own axis once]

3. Accurate estimate of sidereal day:

his estimate:23hrs 56 mins 4.1secs.

current value:23hrs 56 mins 4.091secs.

4. Accurate estimate of sidereal solar year:

His value : 365.25868days

Current value: 365.25636days

5. Introduction of ***Samskara***(reform) :culture of revising data from fresh observations.

6. Conceived ***Gola-yantra-***a spherical apparatus representing the celestial sphere. He prescribed the construction of the sphere. It was designed to rotate uniformly at the rate of Earth’s rotation.

7. Observation of the fact that the cause of shinning of the moon is the reflected sunlight.

**2. Brahmagupta(598-668)ce:** author of **Brahmasphuta–Siddhanta,**which was translated into Arabic in around 771ce and has a major impact on Islamic Mathematics and Astronomy.

His highlights:

Reinforced **aryabhata’s** idea of anther day beginning at midnight.

Gave correct equations for computations of eclipse taking care of the parallax

He theorised that all bodies with mass are attracted to the Earth.

**3. Varahamihira(505ce):**compiled five of the earlier astronomical texts of ancient India called **Siddhantas** (established theories) as **Pancasiddhantika.** Among this, **Surya Siddhanta** is still a popular text among traditional astronomers, especially for computations of calendars.

**4.Bhaskara I(629ce):** composed **Mahabhaskariya, Laghubhaskariya** and **Aryabhatiyabhashya.** The last one is a commentary on **Aryabhatiya.**

**5.Vatesvara(880ce):**author of **Vatasvarasiddhanta**

**6.Lalla(8th century):** author of **Sisyadhivrddhida-**which corrects several assumptions of **Aryabhata.**Here planetary motion calculations along with graphical representation are given

**7.Bhaskara II:** authored **Siddhantasiromoni(1114ce).**he headed the observatory at Ujjain where planetary movements were observed

 **Important Landmarks of Science in ancient India**

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| **Results/Principles** | **Authors and/or Text** | **Adaption/Rediscovery in Europe** |
| Area of a square of diagonal of a rectangle is equal to the sum of the areas on the horizontal and vertical side | Stated in **Sulbasutras of Baudhayana, Apastamba, Katyayana**  (**800bce)** | **Pythagoras(500bce)** |
| To draw a square equal in area to a given rectangle | Known to the authors **of Taittariya Samhita(3000bce)and Satapatha Brahmana(2000bce)** | **Euclid(300bce)** |
| Decimal system(place value and zero)  Concept of powers of10  Zero  Decimal notation | Stated by **Medhatithi in Yajurveda**  Mentioned **by Pingala & Panini**  Anonymous; well known by the time of **Aryabhata(500ce)** | Advocated **by Fibonacci(1202ce)** |
| Binary Representation  **Meru Prstara/Pascal’s Triangle**  Algebraic formula for combinations | **Pingala(3rd century bce),Halayudha(10th -11th century**  **Mahavira(850ce)** | **Leibniz(17th century ce)**  **P Herigone(1634)ascal(1654 ce)** |
| Fundamental Arithmetic operations; fractions, rule of three, commercial arithmetic | Awareness reflected in **Sulvasutras(800bce)Arthasastra(321bce**);well established before **Aryabhata(500ce)** | 15th-16th centuries  (transmitted through Arabs) |
| Basic principles of Algebra; symbols, variables and operations | Systematic exposition **by Brahmagupta(628ce**) | **Viete and others~1600ce** |
| Zero as integer, defined as a-a=0  Negative numbers; rules of operations  ax0=0,(-a)x(+b)=-ab etc | Systematic exposition **by Brahmagupta(628ce)** | **Newton(17th century)** |
| Formation and clearance of equations; solution of quadratic equation; A P Series, G P Series | **Aryabhata(500ce),Brahmagupta(628ce),**  **Sridharacarya(7500ce)** |  |
| Indeterminate (Diophantine)Eqns  Ax-by=c (to be solved in integers  Kuttaka(pulverization principle) | **Sulvasutras(800bce)**  **Aryabhata(500ce)** | **Diophantus(250ce)**  **Bachet(1621ce0**  **Fermat’s descent(17th century)** |
| Geometry of cyclic quadrilaterals; formula for area, diagonals, circumradius | **Brahmagupta(628ce),**  **Paramesvara(15th century)** | **Ptolemy(150ce),Snell(1619),**  **Huler(1782),Fibonacci(1202),**  **Viete(1580),Bachet(1621)** |
| Principle of axial rotation of earth | **Rg Veda, Skanda Purana,**  **Aryabhata(500ce)** | **Copernicus(1543),Galileo(1610)**  **Newton: Theoretical(1687)** |

**SOME ASTRONOMER-MATHMATECIAN AND THEIR MAJOR TEXTS**

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| Author | Texts | Date | Remarks |
| Baudhayana  Apastamba  Katyayana | Sulba-Sutra | 800bce | Oldest extant mathematics text |
| Lagadha | Vedanga -Jyotisa | 1300bce(original)  500bce(present) | Oldest extant astronomy text |
| Ayrabhata | Ayarbhatiya  Aryasiddhanta | 500ce | First astronomy text with a chapter on mathematics  lost |
| Varahamihira  (born 505ce) | Pancasiddhantika  Brhatsamhita |  | Compilations of five earlier siddhanta texts(with refinements) |
| Bhaskara I | Mahabhaskariya  Laghubhaskariya  Aryabhatiya-bhasya | 6th century ce | Important commentary on  Aryabhatiya |
| Brahmagupta | Brahmasphutasiddhana  Khandakhadyaka | 6268ce  655ce | Astronomy text containing a separate chapter on Algebra |
| Sridharacarya | Patiganita | 750ce |  |
| Lalla | Sisyadhivrddhida,  Patiganita, | 768ce | Astronomy test based on Aryabhatiya  lost |
| Mahavira | ganitasarasanigraha | 850ce | Text on arithmetic |
| Prthudakasvamin | Commentary on Brahmasphutasiddhana | 860ce | Endorsed Aryabhata’s principle of axial rotation |
| Gobindasvamin | Govindakrti | 860ce | lost |
| Vatesvara | Vatesvarasiddhanta | 880ce |  |
| Manjula | Laghumanasam | 932ce |  |
| AryabhataII | Mahasiddhanta | 950ce |  |
| Bhaskara II | Siddhantasiromoni,including  Lilavati&Bijaganita | 1150ce |  |
| Narayana Pandita | GanitaKaumudi  Bijaganitavarisa | 1356ce |  |
| Madhavacarya | Venvaroha | 14th century | Original lost, but profusely quoted by later authors |
| Nilakanta | Tantrasamamgraha  Aryabhatiya-bhasya | 1500ce |  |
| Jyesthadeva | Yuktibhasa | 16th century |  |

**For History of mathematics**

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| Author unknown | Bhaksali Manuscript | Uncertain dates,  Probably 3rd -4th century ce | Text on algebra, arithmetic & mensuration |
| Anonymous | Surya Siddhanta | Original-c400ce;  Present 6th-12th century | Popular text on astronomy |
| Medhatithi | Vajasaneyi Samhita  (sukla Yajurveda) |  | Earliest extant exposition of powers of 10 |
| Pingalacarya | Chandas sutra | 3rd century bce | Mention and use of zero; Binary arithmetic; Pascal’ s triangle |
| Kautilya | Arthasastra | 4thcenturybce | Accounting and commercial arithmetic |
| Panini | A treatise of Sanskrit grammar |  | Boolean logic |